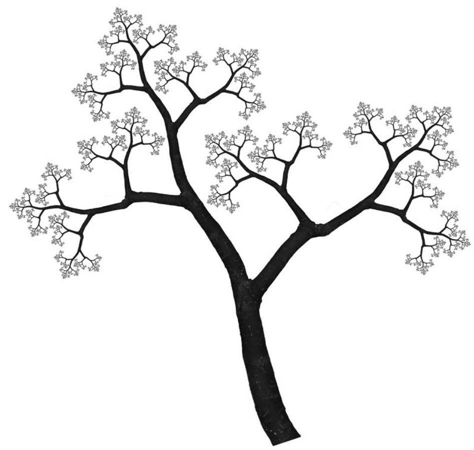
Fractal Trees

Overview

In this lesson, we use the natural fractal branching of a tree to explore quotients and ratios in a simple, tangible way. We also use simple tools like rulers and protractors to measure lengths and angles, seeing how mathematical a complicated tree can be. We investigate naturally occurring cracks outside around your school. This activity wraps up with understanding tree growth and ecology and relationship between tree circumference and age.

Like all fractals, a tree grows by repeating the simple process of branching. This means a little piece of a tree has the same general shape as a bigger part, which also is similar to the entire tree. This property of fractals is known as "self-similarity."



In this activity, we will verify the similarity of branches of a tree to the whole object. To do this, we compare the lengths of connected branches and see that as they get smaller, the ratio of the lengths of branches stays the same.

Appropriate for: grades 3 – 12, college and adult

Objectives

- To characterize trees according to their fractal properties
- To use appropriate tools strategically
- To measure lengths and angles
- To work with quotients, ratios and circumference of a circle0
- To apply mathematical concepts to real-life situations
- To generate and analyze patterns
- To model with mathematics and to classify shapes by properties of their lines and angles
- To attend to precision

Materials

- Protractor
- Ruler with millimeters
- Calculator
- Fractal Tree worksheet
- Pencil

Fractal Trees

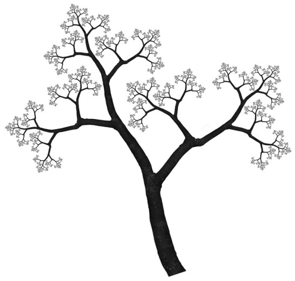
Common Core Standards for Mathematics

Code	Standard	Grade	Code	Standard	Grade
OA	Operations and Algebraic Thinking	4, 5	NS	Number System	6, 7
MD	Measurement and Data	4, 5	RP	Ratios and Proportional Relationships	6, 7
G	Geometry	4, 5, 7	EE	Expressions and Equations	7
NF	Number and Operations – Fractions	4, 5			

HS: Numbers (Q), Modeling, Geometry (SRT, MG)

Common Core Standards for English Language Arts

Code	Standard	Grades K – 5	Grades 6 – 8	Grades 9 – 12
RL	Reading: Literature	1, 4, 7, 10	1, 4, 7, 10	1, 4, 10
RI	Reading: Informational Text	1, 3, 4, 7, 10	1, 3, 4, 7, 10	1, 3, 4, 10
FS	Foundational Skills	1, 2, 3 for grades K – 1; 3 and 4 for grades 2 – 5	None available	None available
W	Writing	2, 3, 8; 4 for grades 3 – 5	2, 3, 4	2, 3, 4, 9
SL	Speaking and Listening	1, 2, 3, 4, 5, 6	1, 2, 3, 4, 5, 6	1, 2, 3, 4, 5, 6
L	Language	1, 4, 6; 3 for grades 2 – 5	1, 3, 4, 6	1, 3, 4, 6
RST	Science and Technical Subjects	None available	1, 3, 4, 6, 7, 10	2, 3, 4, 6, 7, 10



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Fractal Trees

1. How is a tree a fractal?

2. What type of fractal pattern is a tree?

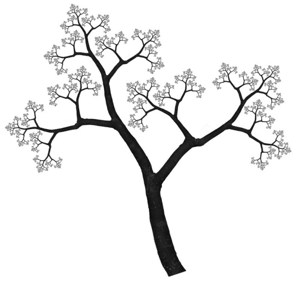
3. List three other naturally-occurring things that are the same type of fractal pattern as a tree.

a) b) c)

4. Use a ruler to measure the distance in millimeters between the bottom of the tree (marked A) and the first branching point (B). Record your measurement in the table under "Distance".

Do this for all sections of the tree.

Section	**Name of Section**	**Distance (mm)**	**Quotient of Adjacent Sections**	**Ratio of Adjacent Sections**
A to B	AB		Example: AB/BC = 64/45 = 1.4	Example: 1.4:1
B to C	BC			
C to D	CD			
D to E	DE			
E to F	EF			
F to G	FG			
G to H	GH			



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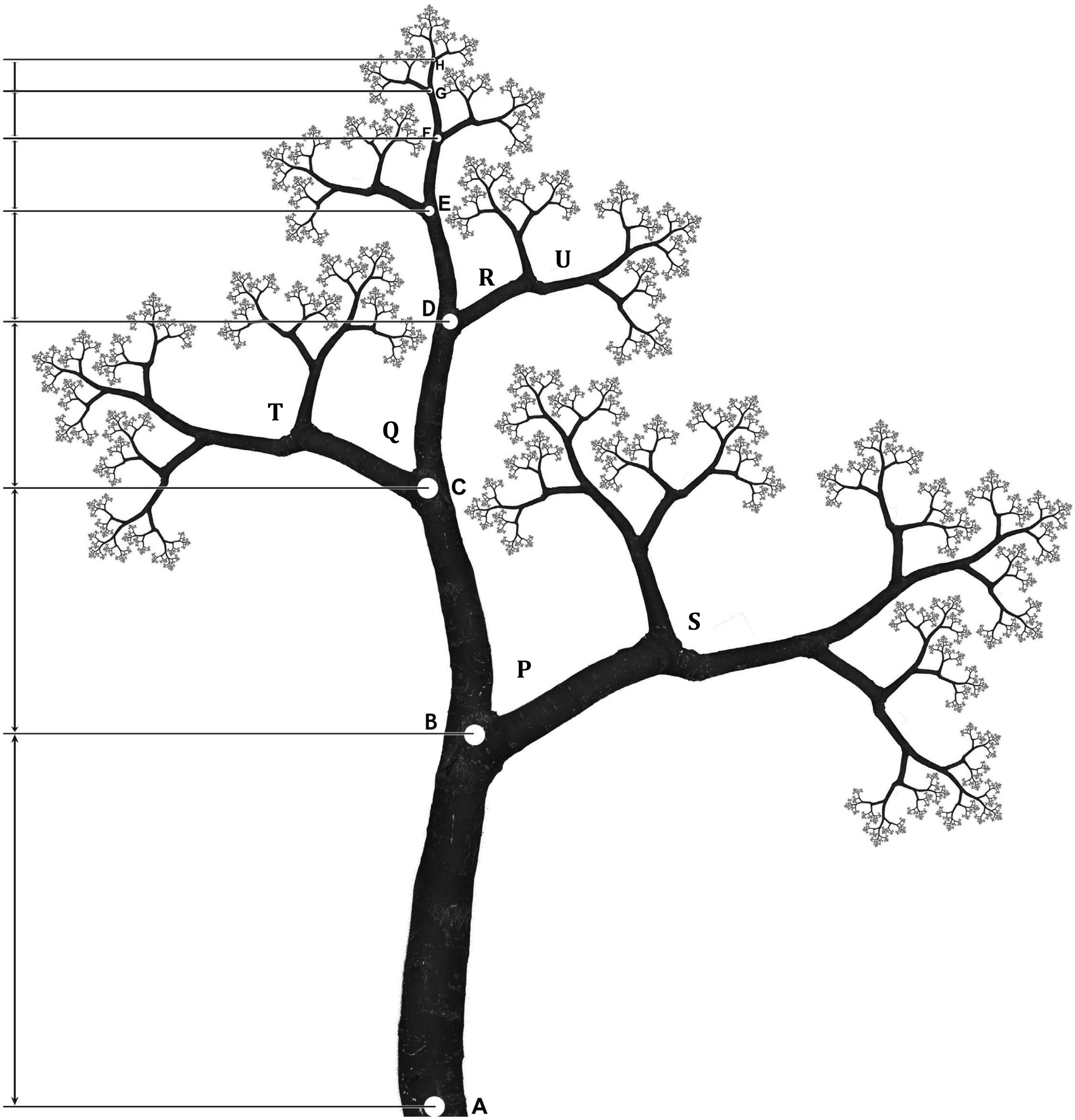
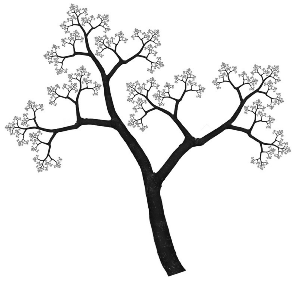
Fractal Trees





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Fractal Trees

Next, we want to see how the size between branches compares from one branch to the next. To do so, we calculate the *quotient* between the lengths of the branches. This is easy to do but will require a calculator.

5. Under "Quotient of Adjacent Sections," write the length of one branch, for instance AB, divided by the length of the next branch, BC, and do the math. The quotient tells us how much bigger the branch is than the next smaller branch. So if AB were twice as long as BC, the quotient would be 2.

6. Finally, write the ratio of the distances. In the case above, with AB and BC, the ratio would be 2:1. If AB were only one and a half times bigger than BC, the quotient would be 1.5, and ratio would be 1.5:1, or 3:2.

Use your calculator to compute the rest of the quotients in the table.

7. What pattern do you see?

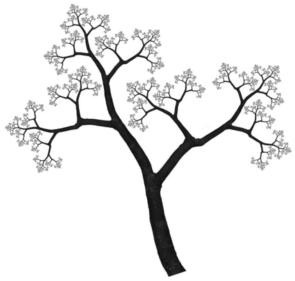
8. Is it easier to measure the branch distances in millimeters instead of inches? If so, why?

9. Use a protractor to measure the angles between the branches. Measure the angles (P, Q, R, S, T and U) and fill in the values in the table to the right.

a) What do you notice about the various angles in the tree?

b) How many kinds of angles can you find in the tree?

Label	Angle
P	
Q	
R	
S	
T	
U	



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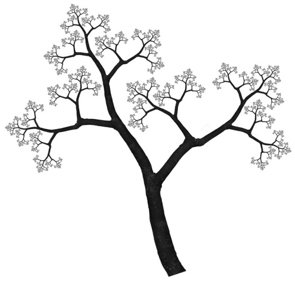
Fractal Trees

Go outside and measure the angles of the cracks around your school. Investigate six of them and draw them, noting the angles and anything else that may affect the cracks, such as the corner of a building or edge of the concrete.

a	b	c
d	e	f

10. Do you notice any patterns in the cracks? If so, what do you see?

Discuss this with your classmates.



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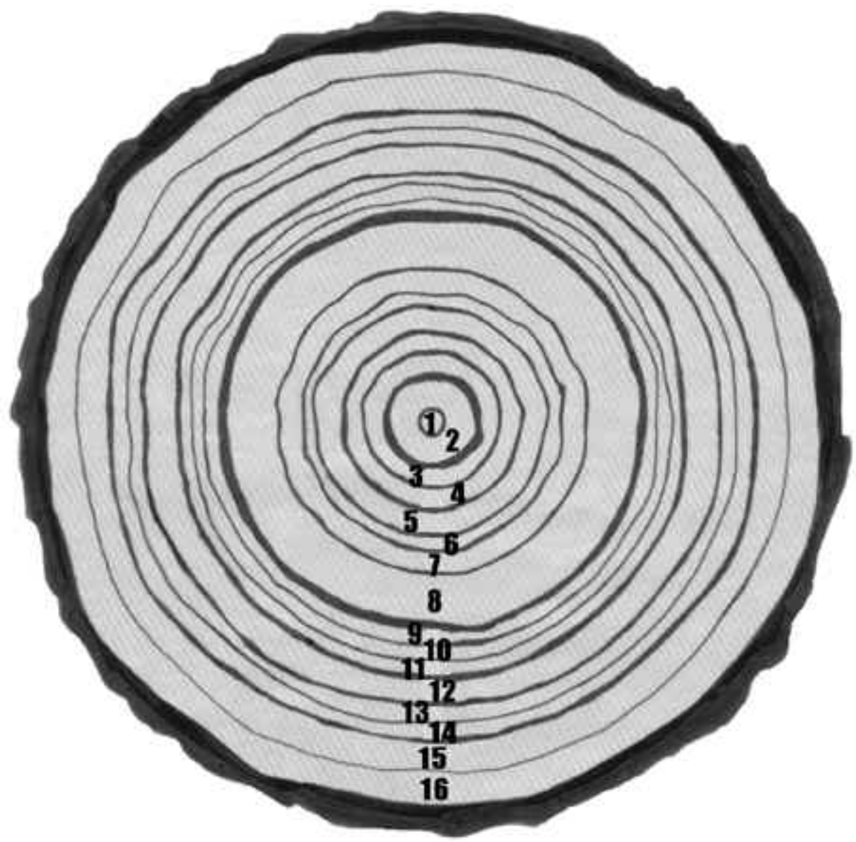
Fractal Trees



Now let's switch gears and look at tree growth. You can tell the age of a tree by counting its rings.

11. How old is the tree pictured here?

12. a) What do you think it means when the rings are really wide?

b) During what two years did the tree grow a lot?

13. a) What were the ecological conditions when the rings are really close together?

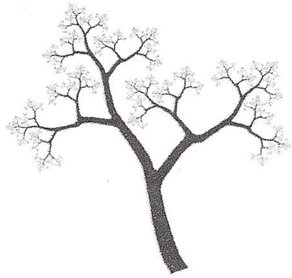
b) When were three consecutive very dry years for this tree?

Biologists measure tree's circumference, which is related to the tree's age. Say the diameter of the tree is 24 centimeters. Draw this on the image above.

14. a) What is the radius?

b) What is the circumference?

15. Using the relationship between circumference and age of the tree above, what would be the estimated age of a tree whose circumference were 95? What is this tree's diameter?



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Fractal Trees

1. How is a tree a fractal?

It is made of lots of simple branching patterns - makes a complicated tree.

2. What type of fractal pattern is a tree?

branching

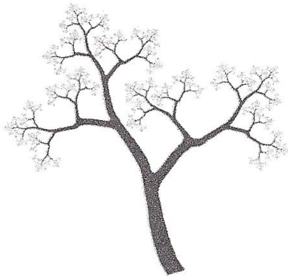
3. List three other naturally-occurring things that are the same type of fractal pattern as a tree.

a) veins　　b) rivers　　c) sidewalk cracks creases on your palms

4. Use a ruler to measure the distance in millimeters between the bottom of the tree (marked A) and the first branching point (B). Record your measurement in the table under "Distance".

Do this for all sections of the tree.

Section	Name of Section	Distance (mm)	Quotient of Adjacent Sections	Ratio of Adjacent Sections
A to B	AB	65	Example: ~~distance~~ 65/45 ~~AB/BC~~ = ~~2~~ 1.4	Example: 2:1
B to C	BC	45	45/30 = 1.5	3:2
C to D	CD	30	30/20 = 1.5 →	3:2
D to E	DE	20	20/15 = 1.3	2:1.5 = 4:3
E to F	EF	15	15/10 = 1.5	3:2
F to G	FG	10	10/5 = 2	2:1
G to H	GH	5		



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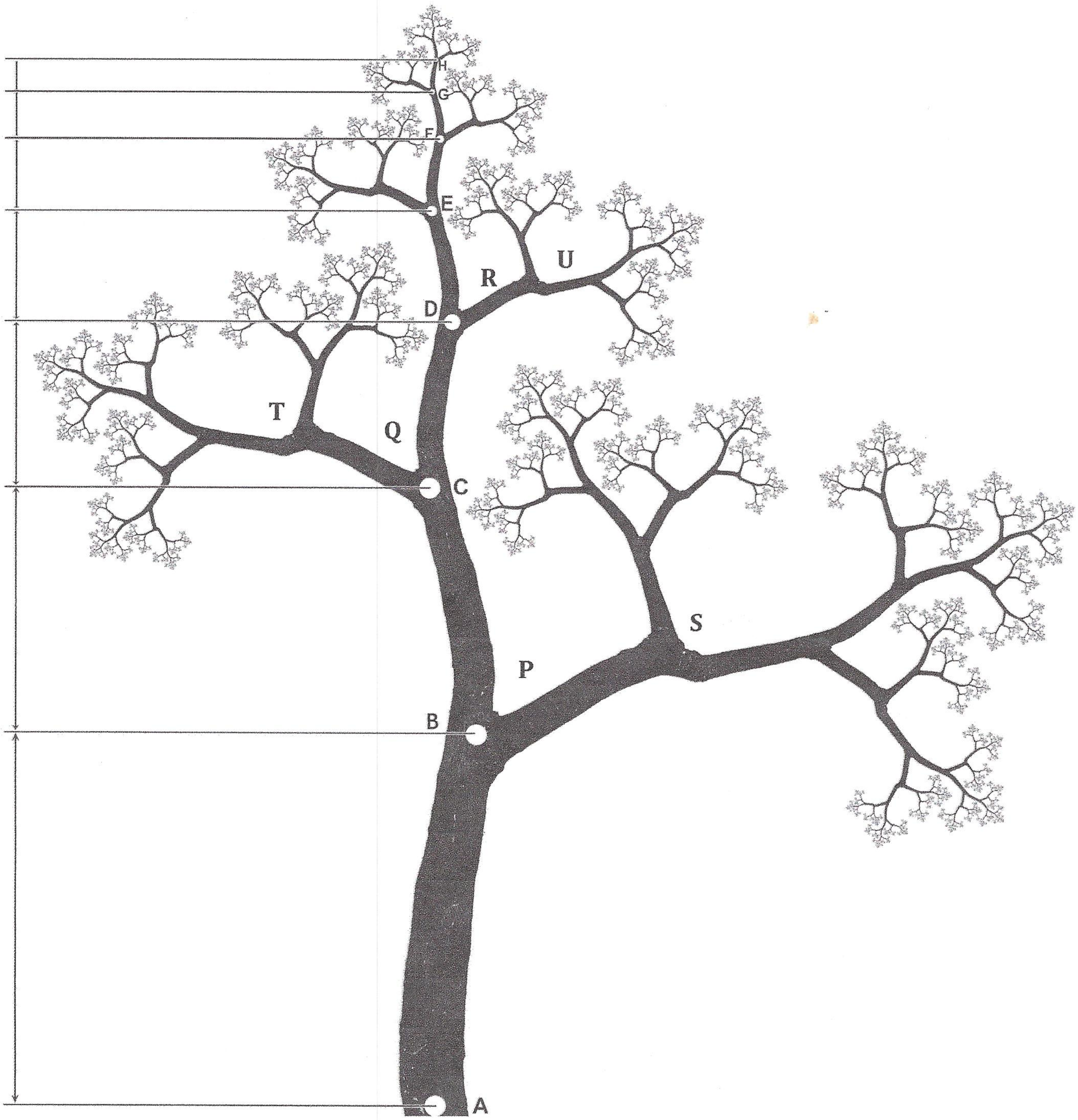

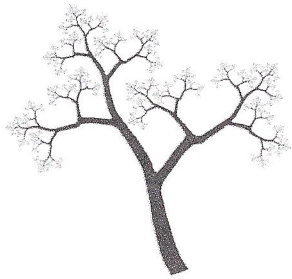

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5. Under "Quotient of Adjacent Sections," write the length of one branch, for instance AB, divided by the length of the next branch, BC, and do the math. The quotient tells us how much bigger the branch is than the next smaller branch. So if AB were twice as long as BC, the quotient would be 2.

6. Finally, write the ratio of the distances. In the case above, with AB and BC, the ratio would be 2:1. If AB were only one and a half times bigger than BC, the quotient would be 1.5, and ratio would be 1.5:1, or 3:2.

Use your calculator to compute the rest of the quotients in the table.

7. What pattern do you see? It's between 1 + 2.

8. Is it easier to measure the branch distances in millimeters instead of inches? If so, why?

millimeters are smaller and more exact

9. Use a protractor to measure the angles between the branches. Measure the angles (P, Q, R, S, T and U) and fill in the values in the table to the right.

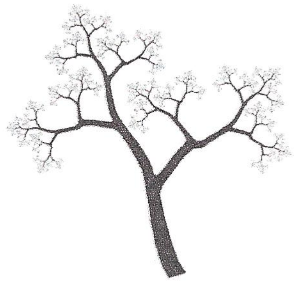
a) What do you notice about the various angles in the tree?

either 45° or 90°

b) How many kinds of angles can you find in the tree?

two - surprising because it seems so complex

Label	Angle
P	45°
Q	45°
R	45°
S	90°
T	90°
U	90°



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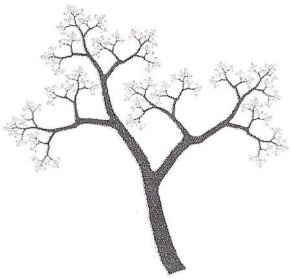
a	b	c
d	e	f

10. Do you notice any patterns in the cracks? If so, what do you see?

Discuss this with your classmates.



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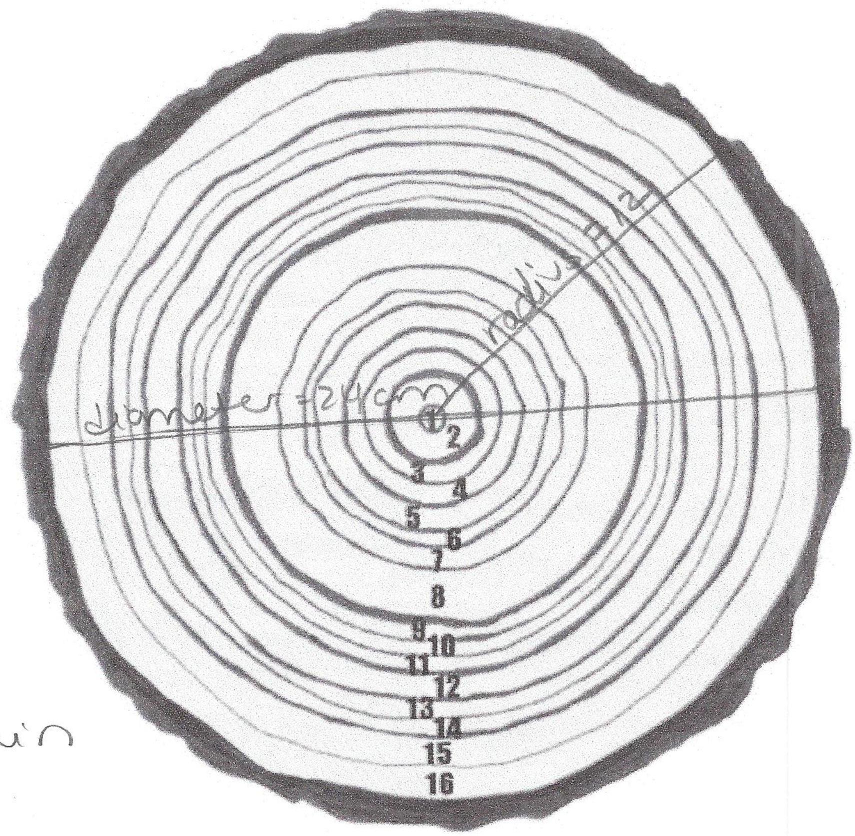
Now let's switch gears and look at tree growth. You can tell the age of a tree by counting its rings.



11. How old is the tree pictured here? 16 years old

12. a) What do you think it means when the rings are really wide?
tree grew well that year - lots of rain

b) During what two years did the tree grow a lot?
years 2 and 8

13. a) What were the ecological conditions when the rings are really close together?
dry / Little to no rain

b) When were three consecutive very dry years for this tree? years 9, 10 and 11

Biologists measure tree's circumference, which is related to the tree's age. Say the diameter of the tree is 24 centimeters. Draw this on the image above.

14. a) What is the radius? 24 cm / 2 = 12 cm

b) What is the circumference? $\pi \times 24\,cm = 3.14 \times 24\,cm = 75.36\,cm$

15. Using the relationship between circumference and age of the tree above, what would be the estimated age of a tree whose circumference were 95? What is this tree's diameter?

$$\frac{75.36\,cm}{16\,years} = \frac{95\,cm}{X\,years} \quad (95 \times 16) \div 75.36 = 20.2 \text{ years old}$$

circumfrence = π x diameter

95 = π x diameter

diameter = $\frac{95}{3.14}$ = 30.25 cm